

Massachusetts Tests for Educator Licensure™

FIELD 47: MIDDLE SCHOOL MATHEMATICS TEST OBJECTIVES

Subarea

	Multiple-Choice	Range of Objectives	Approximate Test Weighting
I.	Number Sense and Operations	01–04	15%
II.	Patterns, Relations, and Algebra	05–10	25%
III.	Geometry and Measurement	11–15	18%
IV.	Data Analysis, Statistics, and Probability	16–17	12%
V.	Trigonometry, Calculus, and Discrete Mathematics	18–20	<u>10%</u>
			80%
	Open-Response		
VI.	Integration of Knowledge and Understanding	21	20%

Copyright © 2002 by National Evaluation Systems, Inc. (NES®)

"Massachusetts Tests for Educator Licensure" and "MTEL" are trademarks of the Massachusetts Department of Education and National Evaluation Systems, Inc. (NES®).

"NES®" and its logo are registered trademarks of National Evaluation Systems, Inc.™

This document may not be reproduced for commercial use but may be copied for educational purposes.

Massachusetts Tests for Educator Licensure™
Test Objectives
Field 47: Middle School Mathematics

SUBAREAS:

NUMBER SENSE AND OPERATIONS
PATTERNS, RELATIONS, AND ALGEBRA
GEOMETRY AND MEASUREMENT
DATA ANALYSIS, STATISTICS, AND PROBABILITY
TRIGONOMETRY, CALCULUS, AND DISCRETE MATHEMATICS
INTEGRATION OF KNOWLEDGE AND UNDERSTANDING

NUMBER SENSE AND OPERATIONS [15%]

0001 Understand the structure of numeration systems and multiple representations of numbers.

For example: place value; number bases (e.g., base 2, base 10); order relations; relationships between operations (e.g., multiplication as repeated additions); number factors and divisibility; prime and composite numbers; prime factorization; multiple representations of numbers (e.g., physical models, diagrams, numerals); and properties of early numeration systems (e.g., Mayan, Mesopotamian, Egyptian).

0002 Understand principles and operations related to integers, fractions, decimals, percents, ratios, and proportions.

For example: order of operations; identity and inverse elements; associative, commutative, and distributive properties; absolute value; operations with signed numbers; multiple representations (e.g., area models for multiplication) of number operations; analyzing standard algorithms for addition, subtraction, multiplication, and division of integers and rational numbers; number operations and their inverses; and the origins and development of standard computational algorithms.

0003 Understand and solve problems involving integers, fractions, decimals, percents, ratios, and proportions.

For example: solving a variety of problems involving integers, fractions, decimals, percents (including percent increase and decrease), ratios, proportions, and average rate of change; and using estimation to judge the reasonableness of solutions to problems.

**Field 47: Middle School Mathematics
Test Objectives**

0004 Understand the properties of real numbers and the real number system.

For example: rational and irrational numbers; properties (e.g., closure, distributive, associative) of the real number system and its subsets; operations and their inverses; the real number line; roots and powers; the laws of exponents; scientific notation; using number properties to prove theorems (e.g., the product of two even numbers is even); and problems involving real numbers and their operations.

PATTERNS, RELATIONS, AND ALGEBRA [25%]

0005 Understand and use patterns to model and solve problems.

For example: making conjectures about patterns presented in numeric, geometric, or tabular form; representing patterns and relations using symbolic notation; identifying patterns of change created by functions (e.g., linear, quadratic, exponential); and using finite and infinite series and sequences (e.g., Fibonacci, arithmetic, geometric) to model and solve problems.

0006 Understand how to manipulate and simplify algebraic expressions and translate problems into algebraic notation.

For example: the nature of a variable; evaluating algebraic expressions for a given value of a variable; the relationship between standard computational algorithms and algebraic processes; expressing direct and inverse relationships algebraically; expressing one variable in terms of another; manipulating and simplifying algebraic expressions; solving equations; and using algebraic expressions to model situations.

0007 Understand properties of functions and relations.

For example: the difference between functions and relations; the generation and interpretation of graphs that model real-world situations; multiple ways of representing functions (e.g., tabular, graphic, verbal, symbolic); properties of functions and relations (e.g., domain, range, continuity); piecewise-defined functions; addition, subtraction, and composition of functions; and graphs of functions and their transformations [e.g., the relationships among $f(x)$, $f(x + k)$, and $f(x) + k$].

0008 Understand properties and applications of linear relations and functions.

For example: the relationship between linear models and rate of change; direct variation; graphs of linear equations; slope and intercepts of lines; finding an equation for a line; methods of solving systems of linear equations and inequalities (e.g., graphing, substitution); and modeling and solving problems using linear functions and systems.

**Field 47: Middle School Mathematics
Test Objectives**

0009 Understand properties and applications of quadratic relations and functions.

For example: methods of solving quadratic equations and inequalities (e.g., factoring, completing the square, quadratic formula, graphing); real and complex roots of quadratic equations; graphs of quadratic functions; quadratic maximum and minimum problems; and modeling and solving problems using quadratic relations, functions, and systems.

0010 Understand properties and applications of exponential, polynomial, rational, and absolute value functions and relations.

For example: problems involving exponential growth (e.g., population growth, compound interest) and decay (e.g., half-life); inverse variation; modeling problems using rational functions; properties and graphs of polynomial, rational, and absolute value functions; and the use of graphing calculators and computers to find numerical solutions to problems involving exponential, polynomial, rational, and absolute value functions.

GEOMETRY AND MEASUREMENT [18%]

0011 Understand principles, concepts, and procedures related to measurement.

For example: using appropriate units of measurement; unit conversions within and among measurement systems; problems involving length, area, volume, mass, capacity, density, time, temperature, angles, and rates of change; problems involving similar plane figures and indirect measurement; the effect of changing linear dimensions on measures of length, area, or volume; and the effects of measurement error and rounding on computed quantities (e.g., area, density, speed).

0012 Understand the principles of Euclidean geometry and use them to prove theorems.

For example: the nature of axiomatic systems; undefined terms and postulates of Euclidean geometry; relationships among points, lines, angles, and planes; methods for proving triangles congruent; properties of similar triangles; justifying geometric constructions; proving theorems within the axiomatic structure of Euclidean geometry; and the origins and development of geometry in different cultures (e.g., Greek, Hindu, Chinese).

**Field 47: Middle School Mathematics
Test Objectives**

0013 Apply Euclidean geometry to analyze the properties of two-dimensional figures and to solve problems.

For example: using deduction to justify properties of and relationships among triangles, quadrilaterals, and other polygons (e.g., length of sides, angle measures); identifying plane figures given characteristics of sides, angles, and diagonals; the Pythagorean theorem; special right triangle relationships; arcs, angles, and segments associated with circles; deriving and applying formulas for the area of composite shapes; and modeling and solving problems involving two-dimensional figures.

0014 Solve problems involving three-dimensional shapes.

For example: area and volume of and relationships among three-dimensional figures (e.g., prisms, pyramids, cylinders, cones); perspective drawings; cross sections (including conic sections) and nets; deriving properties of three-dimensional figures from two-dimensional shapes; and modeling and solving problems involving three-dimensional geometry.

0015 Understand the principles and properties of coordinate and transformational geometry.

For example: representing geometric figures (e.g., triangles, circles) in the coordinate plane; using concepts of distance, midpoint, slope, and parallel and perpendicular lines to classify and analyze figures (e.g., parallelograms); characteristics of dilations, translations, rotations, reflections, and glide-reflections; types of symmetry; properties of tessellations; transformations in the coordinate plane; and using coordinate and transformational geometry to prove theorems and solve problems.

DATA ANALYSIS, STATISTICS, AND PROBABILITY [12%]

0016 Understand descriptive statistics and the methods used in collecting, organizing, reporting, and analyzing data.

For example: constructing and interpreting tables, charts, and graphs (e.g., line plots, stem-and-leaf plots, box plots, scatter plots); measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, standard deviation); frequency distributions; percentile scores; the effects of data transformations on measures of central tendency and variability; evaluating real-world situations to determine appropriate sampling techniques and methods for gathering and organizing data; making appropriate inferences, interpolations, and extrapolations from a set of data; interpreting correlation; and problems involving linear regression models.

**Field 47: Middle School Mathematics
Test Objectives**

0017 Understand the fundamental principles of probability.

For example: representing possible outcomes for a probabilistic situation; counting strategies (e.g., permutations and combinations); computing theoretical probabilities for simple and compound events; using simulations to explore real-world situations; connections between geometry and probability (e.g., probability as a ratio of two areas); and using probability models to understand real-world phenomena.

TRIGONOMETRY, CALCULUS, AND DISCRETE MATHEMATICS [10%]

0018 Understand the properties of trigonometric functions and identities.

For example: degree and radian measure; right triangle trigonometry; the law of sines and the law of cosines; graphs and properties of trigonometric functions and their inverses; amplitude, period, and phase shift; trigonometric identities; and using trigonometric functions to model real-world periodic phenomena.

0019 Understand the conceptual basis of calculus.

For example: the concept of limit; the relationship between slope and rates of change; how the derivative relates to maxima, minima, points of inflection, and concavity of curves; the relationship between integration and the area under a curve; modeling and solving basic problems using differentiation and integration; and the development of calculus.

0020 Understand the principles of discrete/finite mathematics.

For example: properties of sets; recursive patterns and relations; problems involving iteration; properties of algorithms; finite differences; linear programming; properties of matrices; and characteristics and applications of graphs and trees.

**Field 47: Middle School Mathematics
Test Objectives**

INTEGRATION OF KNOWLEDGE AND UNDERSTANDING [20%]

In addition to answering multiple-choice items, candidates will prepare written responses to questions addressing content from the preceding objectives, which are summarized in the objective and descriptive statement below.

- 0021 Prepare an organized, developed analysis on a topic related to one or more of the following: number sense and operations; patterns, relations, and algebra; geometry and measurement; data analysis, statistics, and probability; and trigonometry, calculus, and discrete mathematics.**

For example: presenting a detailed solution to a problem involving one or more of the following: place value, number base, and the structure and operations of number systems; application of ratios and proportions in a variety of situations; properties, attributes, and representations of linear functions; modeling problems using exponential functions; the derivative as a rate of change and the integral as area under the curve; applications of plane and three-dimensional geometry; and design, analysis, presentation, and interpretation of a statistical survey.