



NATIONAL
Singapore Math
Summer Institute

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MENTAL MATH

Emphasizes an understanding of
place value and the
distributive, commutative and
associative properties



Basic Definitions

Subtraction is the opposite of addition:

Asking the question $15 - 9 = ?$ is the same as asking $? + 9 = 15$

Multiplication is repeated addition:

$$3 \times 5 = 5 + 5 + 5$$

Division is the opposite of multiplication:

Asking the question $18 \div 3 = ?$ is the same as asking $? \times 3 = 18$

Properties of Arithmetic

Commutative: For any two numbers a and b :

$$a + b = b + a \qquad a \times b = b \times a$$

Associative: For any three numbers a , b and c :

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

Properties of Arithmetic

Commutative:

$$3 + 4 = 4 + 3$$

$$3 \times 4 = 4 \times 3$$

Associative:

$$(3 + 4) + 5 = 3 + (4 + 5)$$

$$(3 \times 4) \times 5 = 3 \times (4 \times 5)$$

Properties of Arithmetic

For an elementary school child, these four properties mean that we can add numbers in any order or multiply numbers in any order.

Distributive Property

For any three numbers a, b and c:

$$a \times (b + c) = a \times b + a \times c$$

Example:

$$3 \times 12 = (3 \times 10) + (3 \times 2)$$

Multiplication and Place Value

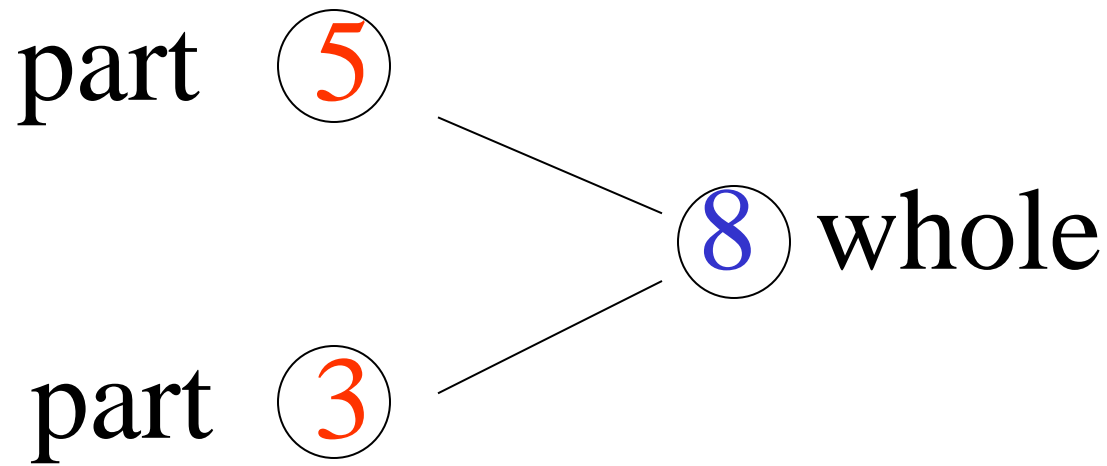
$$3 \times 2 = 3 \times 2 \text{ ones} = 6 \text{ ones}$$

$$3 \times 20 = 3 \times 2 \text{ tens} = 6 \text{ tens}$$

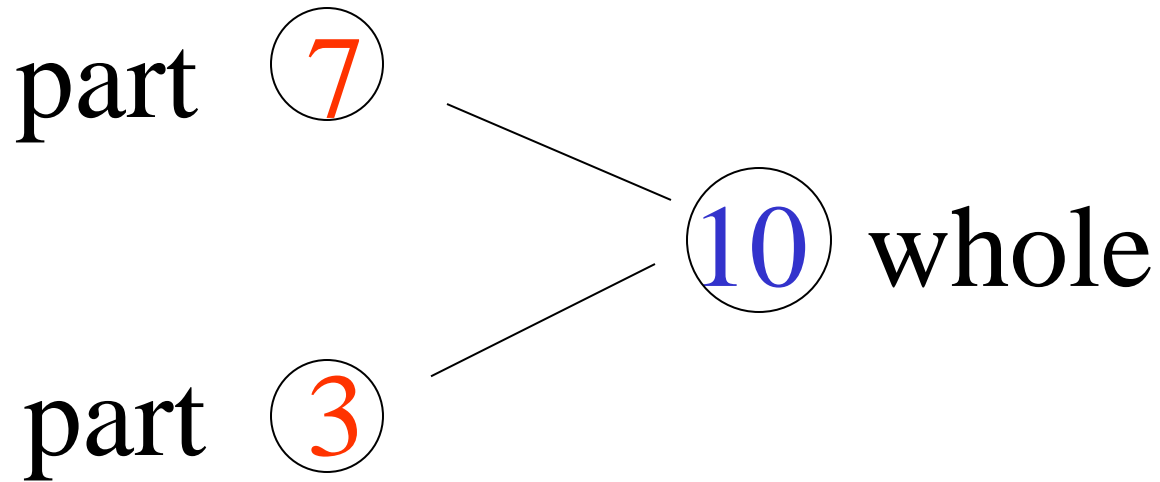
$$3 \times 200 = 3 \times 2 \text{ hundreds} = 6 \text{ hundreds}$$

This is the associative property.

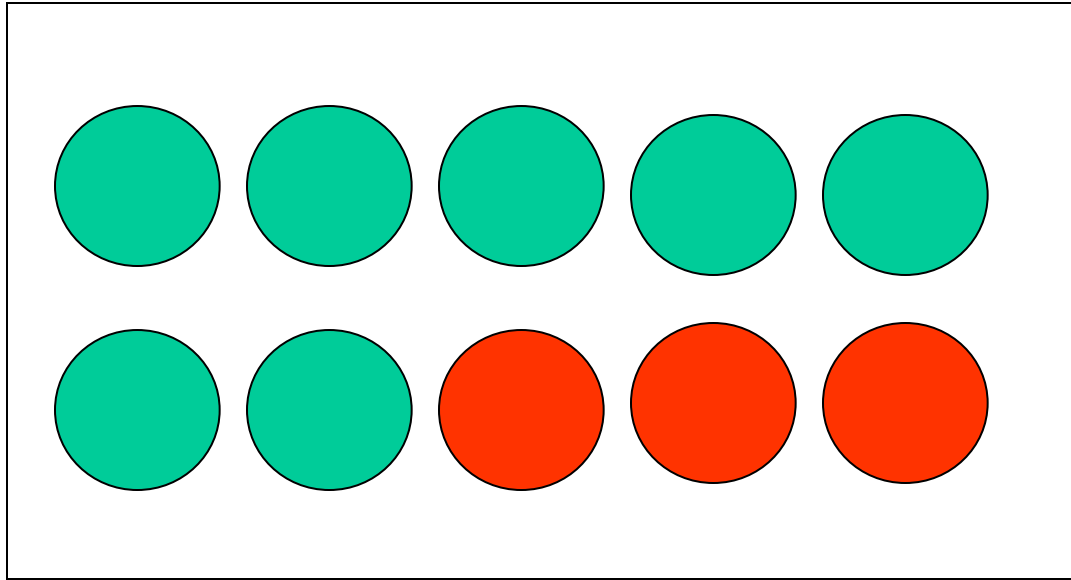
Number Bonds



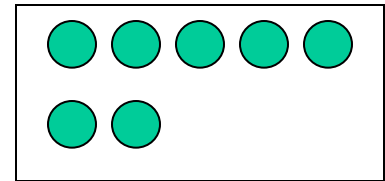
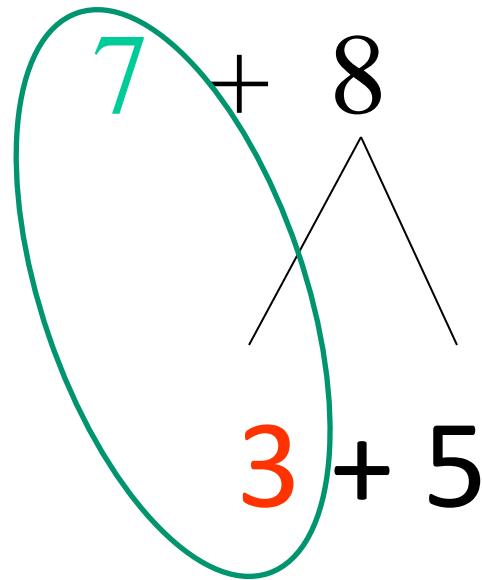
Making Ten



Ten Frame



Compensation



You can add by making a 10.

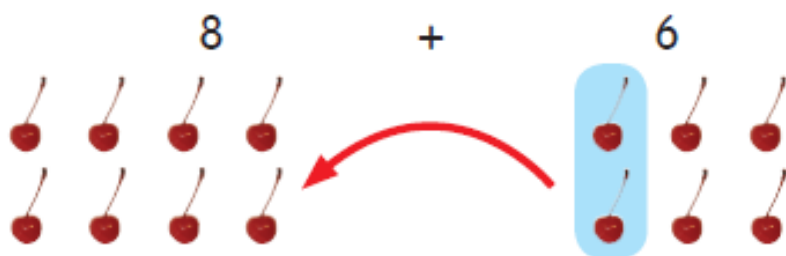
Gus has 8 cherries.

Ava gives him 6 more.

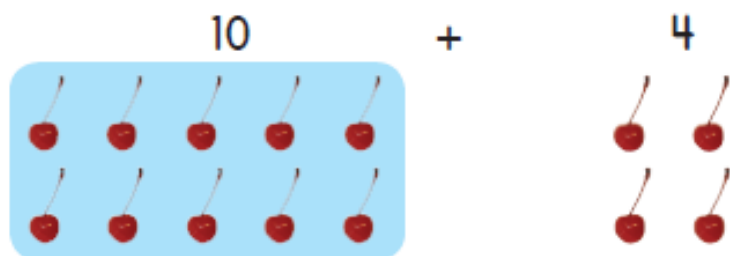


How many cherries does Gus have now?

Step 1 Make a **group** of 10 cherries.



Step 2 Add the cherries that are left over to the group of 10.



$$10 + 4 = 14$$

Gus has 14 cherries now.

You can break the number that is less into 2 parts.

$$8 + 6 = 10 + 4 = 14$$

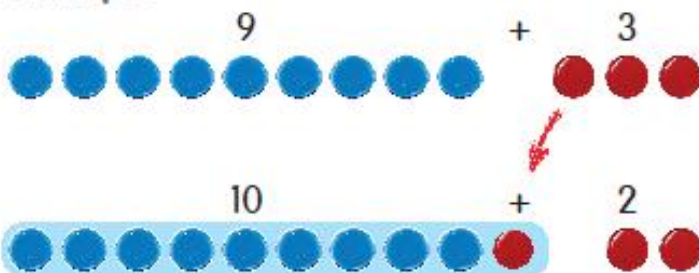



Hands-On Activity

Use .


Group the  to make a 10.
Then add.

Example





$$\begin{aligned} 9 + 3 &= 10 + 2 \\ &= 12 \end{aligned}$$




$$\begin{aligned} 8 + 6 &= 10 + \square \\ &= \square \end{aligned}$$




$$\begin{aligned} 7 + 6 &= 10 + \square \\ &= \square \end{aligned}$$

Let's Practice

Make a 10.
Then add.

1 $9 + 4 =$



2 $7 + 9 =$



3 $9 + 8 =$



4 $8 + 3 = 10 +$
 $=$

5 $6 + 8 =$ +
 $=$

ON YOUR OWN

Go to Workbook A:
Practice 1, pages 191–196

You can use doubles plus one facts to add.

$2 + 2 = 4$ is a doubles fact.



What is $2 + 3$?



You can rewrite $2 + 3$ like this:

$$2 + 2 + 1$$

So, $2 + 3$ is double 2 plus 1.

$2 + 3$ is a **doubles plus one** fact.

You can use the doubles fact $2 + 2$ to add 2 and 3.

$$\underline{2 + 2} = 4$$

doubles

$$2 + 3 = \underline{2 + 2} + 1$$

doubles plus 1

$$= 4 + 1$$

$$= 5$$

2 + 2 and 1 more.



Guided Practice

Solve.

- 8 Which are the doubles facts?
Which are the doubles plus one facts?

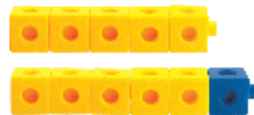
$4 + 4 = 8$

$4 + 5 = 9$

$8 + 7 = 15$

$7 + 7 = 14$

- 9 $5 + 6 = ?$



$5 + 6$ is double plus .

$5 + 6 = 5 + \text{} + \text{$

$= 10 + \text{$

$= \text{$

Let's Practice

Solve.

1 a Double 6 is + =

b $6 + 7 = \text{} + \text{} + \text{$
 $= \text{$

2 a What doubles fact helps you to add 9 and 8?

b $9 + 8 = \text{$

ON YOUR OWN

Go to Workbook A:
Practice 3, pages 199–202

You can subtract by grouping into a 10 and ones.

Shawn makes 12 stars.

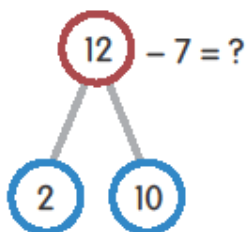
He gives 7 to Gina.

How many stars does Shawn have left?



Step 1 Group 12 into a 10 and ones.

$$12 = 10 + 2$$



Step 2 You cannot subtract 7 from 2.

So, subtract 7 from 10.

$$10 - 7 = 3$$

Step 3 Add the ones.

$$2 + 3 = 5$$

So, $12 - 7 = 5$.

Shawn has 5 stars left.

LESSON 6

Divide Using Related Multiplication Facts

Lesson Objectives

- Use related multiplication facts to find related division facts.
- Write a multiplication sentence and a related division sentence.
- Solve division word problems.

Learn You can use related multiplication facts to help you divide when you share equally.

Divide 12 sharpeners into 2 equal groups.
How many sharpeners are in each group?



$$12 \div 2 = ?$$

6 sharpeners are in each group.

$$6 \times 2 = 12$$

So, $12 \div 2 = 6$.



Guided Practice

Find the missing numbers.

Use related multiplication facts to help you divide.

- 1 5 children share 10 cherries equally.
How many cherries does each child get?



$$10 \div 5 = 2$$

Each child gets 2 cherries.

$$2 \times 5 = 10$$

So, $10 \div 5 = 2$.



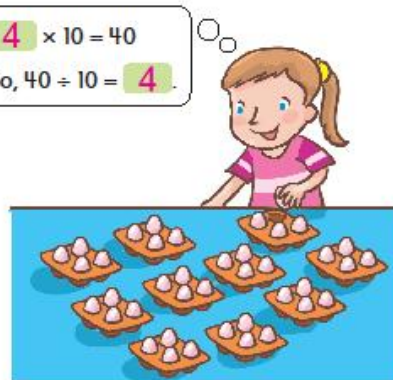
- 2 Kelly puts 40 eggs equally on 10 trays.
How many eggs are on each tray?

$$40 \div 10 = 4$$

4 eggs are on each tray.

$$4 \times 10 = 40$$

$$\text{So, } 40 \div 10 = 4.$$



Learn You can use related multiplication facts to help you divide when you put things in equal groups.



Divide 35 cubes into equal groups.
There are 5 cubes in each group.
How many groups are there?

$$35 \div 5 = ?$$

There are 7 groups.

$$7 \times 5 = 35$$
$$\text{So, } 35 \div 5 = 7.$$



More Compensation - Addition

$$\begin{aligned} 99 + 28 &= 99 + (1 + 27) \\ &= (99 + 1) + 27 \end{aligned}$$

$$\begin{aligned} 43 + 78 &= (41 + 2) + 78 \\ &= 41 + (2 + 78) \\ &= 41 + 80 = 1 + 40 + 80 \end{aligned}$$

Subtraction Compensation

$$72 - 59 = ?$$

Imagine a number line

The distance for 59 to 72 is the same as the distance from 60 to 73

$$\text{So } 72 - 59 = 73 - 60$$

Add 1 to each

Can you do it another way imagining a number line?

Try $93 - 18$

Multiplication Compensation

$$50 \times 18$$

(how much money is 18 fifties?)

$$= 50 \times (2 \times 9)$$

$$= (50 \times 2) \times 9$$

$$= 100 \times 9 = 900$$

Division Compensation

$$2300 \div 50 =$$

$$4600 \div 100$$

Double each.

WHY????

Multiplying or Dividing by 5

- Multiply by 5
Halve and multiply by 10:

$$62 \times 5$$

- Divide by 5
Divide by 10 and double:

$$142 \div 5$$

Multiplying or Dividing by 4

- Multiply by 4

Double twice: 26×4

- Divide by 4

Halve twice: $184 \div 4$

Multiplying by 9

Multiply by 10 and subtract the original number:

$$35 \times 9 =$$

$$(35 \times 10) - (35 \times 1)$$

Multiply by 25

$$48 \times 25$$



Multiply by 25

$$48 \times 25$$

$$= (12 \times 4) \times 25$$

$$= 12 \times (4 \times 25)$$

$$= 12 \times 100$$

Quarter and multiply by 100

Distributive Property (again)

$$13 \times 98$$

$$= (13 \times 100) - (13 \times 2)$$

$$= 1300 - 26$$

Various Properties

that let us rearrange

$$2 \times 387 \times 5 = (2 \times 5) \times 387$$

$$256 \times 4 = (250 \times 4) + (6 \times 4)$$

What are the formal names for each property?

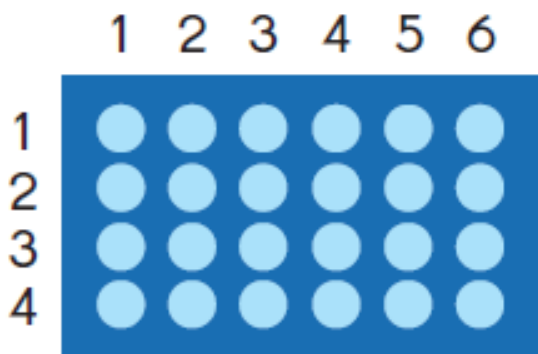
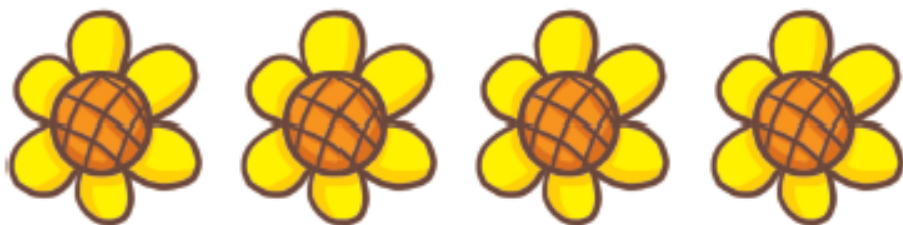
Math In Focus

Examples

Note – Mastery
through
understanding

2B & 3A

2 A flower has 6 petals.
Julia buys 4 flowers.
How many petals does Julia have in all?

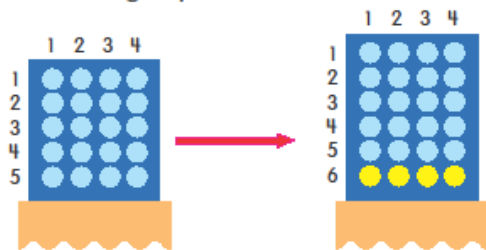


$$4 \times 6 = \square \times \square$$
$$= \square$$

Julia has petals in all.

Learn You can use multiplication facts you know to find other multiplication facts.

$6 \times 4 = ?$
Start with 5 groups of 4.



6×4 is the same as adding 1 group of 4 to 5×4 .



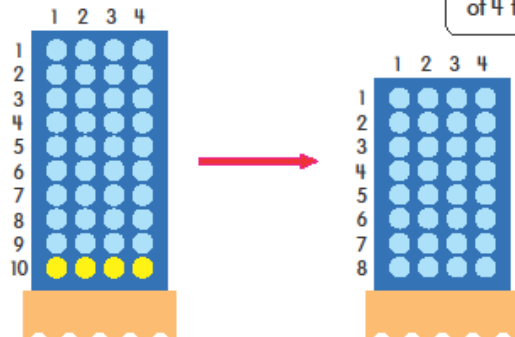
$5 \times 4 = 20$

$6 \times 4 = 5 \text{ groups of } 4 + 1 \text{ group of } 4$
 $= 20 + 4$
 $= 20$

Guided Practice

Use facts you know to find the missing numbers.

2 $8 \times 4 = ?$
Start with 10 groups of 4.



8×4 is the same as subtracting groups of 4 from 10×4 .



$10 \times 4 = \text{$

$8 \times 4 = 10 \text{ groups of } 4 - \text{$ groups of 4
 $= \text{$ - $\text{$ = $\text{$

Let's Practice

Find the missing numbers.

1 $5 \times 4 = \square$

$$\begin{aligned} 6 \times 4 &= 5 \text{ groups of } 4 + \square \text{ group of } 4 \\ &= \square + 4 \\ &= \square \end{aligned}$$

$$\begin{aligned} 7 \times 4 &= 5 \text{ groups of } 4 + \square \text{ groups of } 4 \\ &= \square + \square \\ &= \square \end{aligned}$$

2 $10 \times 4 = \square$

$$\begin{aligned} 9 \times 4 &= 10 \text{ groups of } 4 - \square \text{ group of } 4 \\ &= \square - 4 \\ &= \square \end{aligned}$$

$$\begin{aligned} 8 \times 4 &= 10 \text{ groups of } 4 - \square \text{ groups of } 4 \\ &= \square - \square \\ &= \square \end{aligned}$$

Use dot paper to find the missing numbers.

3 $3 \times 4 = \square$

$4 \times 3 = \square$

4 $6 \times 4 = \square$

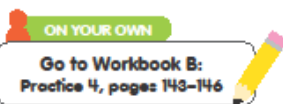
$4 \times 6 = \square$

5 $5 \times 4 = \square$

$4 \times 5 = \square$

6 $8 \times 4 = \square$

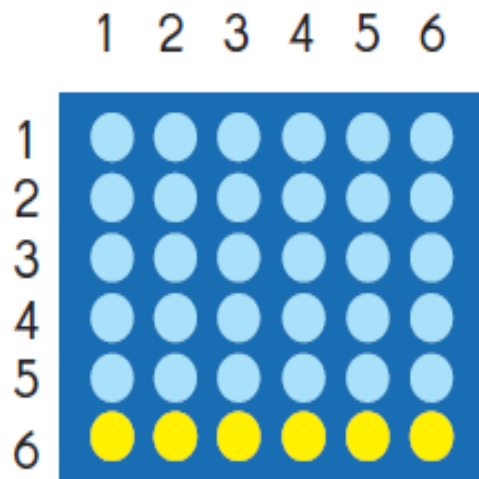
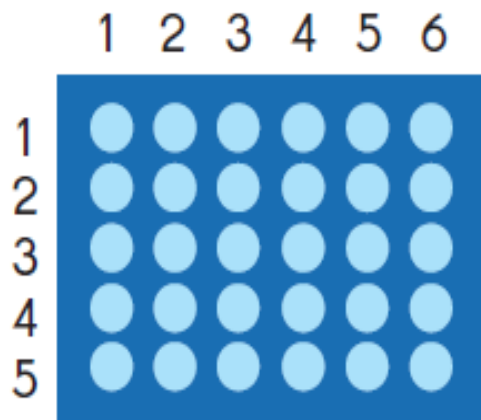
$4 \times 8 = \square$



Find the missing numbers. Use array models to help you.

3 $6 \times 6 = ?$

Start with 5 groups of 6.



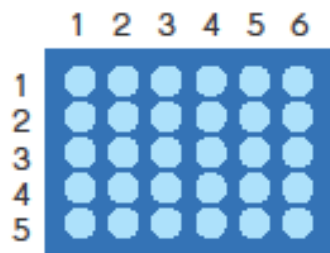
$$5 \times 6 = \square$$

$$\begin{aligned} 6 \times 6 &= 5 \text{ groups of } 6 \\ &+ \square \text{ group of } 6 \\ &= 30 + \square \\ &= \square \end{aligned}$$

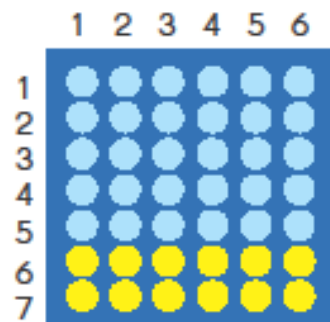
Use multiplication facts you know to find other multiplication facts.

$$7 \times 6 = ?$$

Start with 5 groups of 6.



$$5 \times 6 = 30$$



$$\begin{aligned} 7 \times 6 &= 5 \text{ groups of } 6 \\ &\quad + 2 \text{ groups of } 6 \\ &= 30 + 12 \\ &= 42 \end{aligned}$$

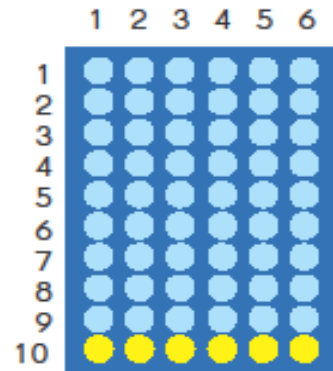
$$\begin{aligned} 5 \times 6 &= 6 \times 5 \\ &= 30 \end{aligned}$$

7×6 is the same as adding 2 groups of 6 to 5 groups of 6.

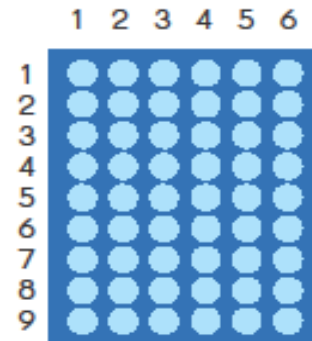


$$9 \times 6 = ?$$

Start with 10 groups of 6.



$$10 \times 6 = 60$$



$$\begin{aligned} 9 \times 6 &= 10 \text{ groups of } 6 \\ &\quad - 1 \text{ group of } 6 \\ &= 60 - 6 \\ &= 54 \end{aligned}$$

$$\begin{aligned} 10 \times 6 &= 6 \times 10 \\ &= 60 \end{aligned}$$

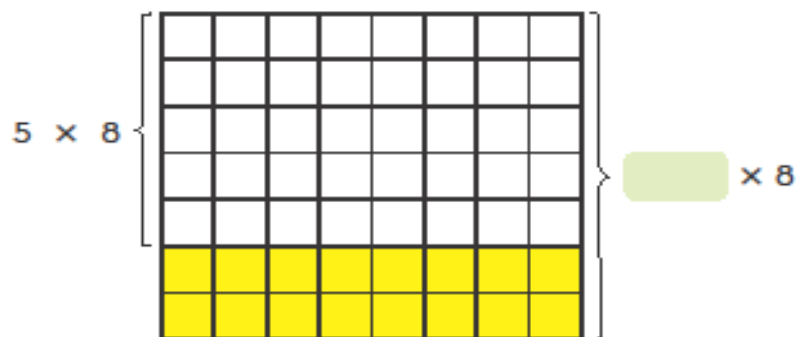
9×6 is the same as
subtracting 1 group of 6
from 10×6 .



Guided Practice

Solve. Use area models to help you.

- 3** Lily has 7 packs of paper plates.
Each pack contains 8 paper plates.
How many paper plates does Lily have in all?



$$\begin{aligned} 7 \times 8 &= 5 \text{ groups of } 8 + \text{ [] groups of } 8 \\ &= 40 + \text{ []} \\ &= \text{ []} \end{aligned}$$

Lily has [] paper plates in all.

$$\begin{aligned} 5 \times 8 &= 8 \times 5 \\ &= 40 \end{aligned}$$

7×8 is the same as adding 2 groups of 8 to 5×8 .





Mental Math Problems - I

1. $95 + 47 =$ _____

2. $81 - 38 =$ _____

3. $268 - 69 =$ _____

4. $362 - 98 =$ _____





Mental Math Problems - II

5. $50 \times 42 =$ _____

6. $25 \times 64 =$ _____

7. $20 \times 11 =$ _____

8. $50 \times 19 =$ _____





Mental Math Problems - III

9. $1600 \div 50 =$ _____

10. $325 \div 50 =$ _____

11. $1250 \div 25 =$ _____

12. $60 \div 25 =$ _____

13. $12 \div 20 =$ _____

14. $7 \div 25 =$ _____





Mental Math Problems - IV

15. $210 \div 20 =$ _____

16. $15 \times 9 =$ _____

17. $15 \times 19 =$ _____

18. $51 \times 38 =$ _____

19. $28 \times 99\frac{5}{7} =$ _____

