

***Visual Manipulatives:
Create Mathematical
Coherence for K-8
Students Using Model
Drawings***

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PROBLEM SOLVING WITH MODEL DRAWING

The model drawing approach takes students from the concrete to the abstract stage via an intermediary pictorial stage.

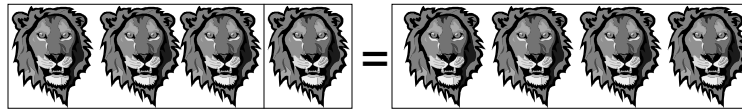
Students create bars and break them down into “units.” The units create a bridge to the concept of an “unknown” quantity that must be found.

Students can learn to use this strategy in the primary grades and continue with it through the middle grades.

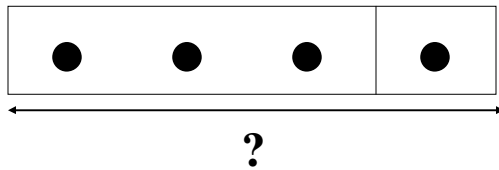
Model Drawing Stages



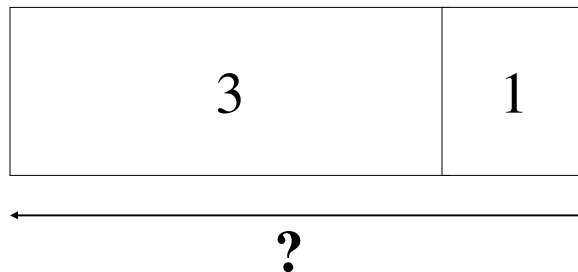
2. Draw pictures inside a bar



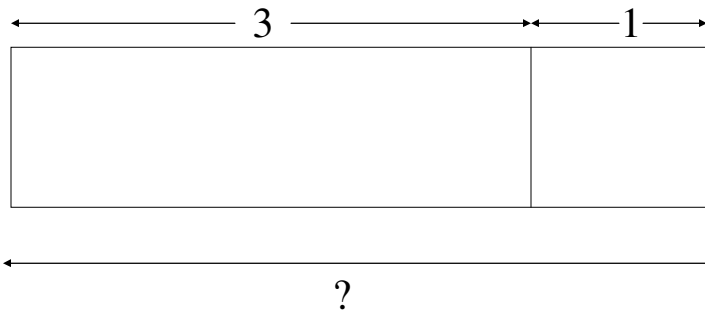
3. Replace pictures with dots and draw arrows outside bars



4. Use numbers inside the bars

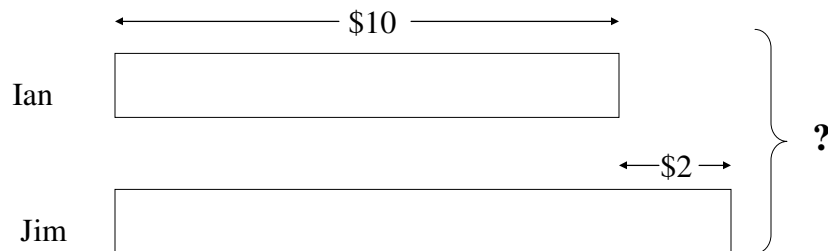


5. Use numbers outside the bars

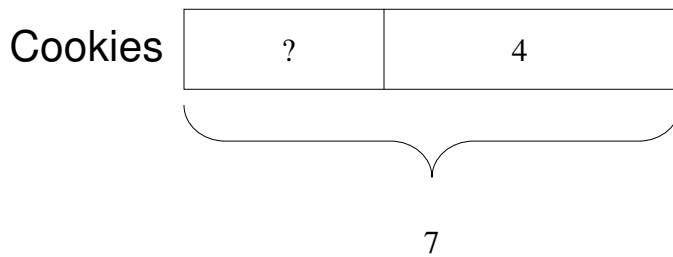


6. Label the bars.

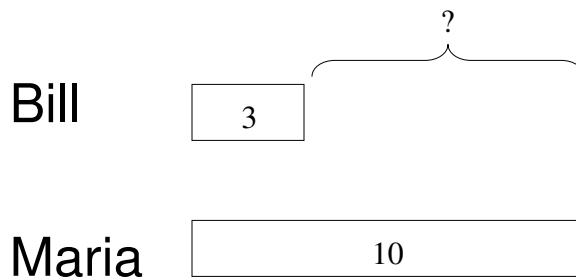
Example: Ian has \$10. Jim has \$2 more than Ian. How much do they have altogether.



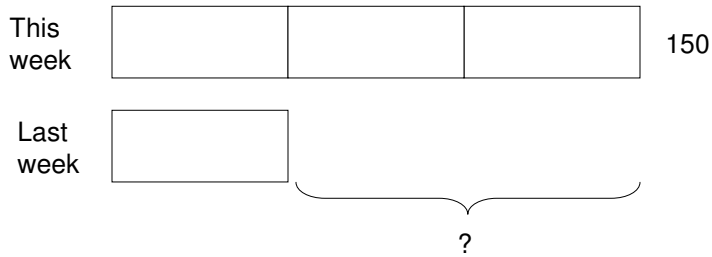
Bill has 7 cookies. He eats 4 cookies.
How many cookies remain?



Maria has 10 cookies. Bill has 3
cookies. How many more cookies
does Maria have?



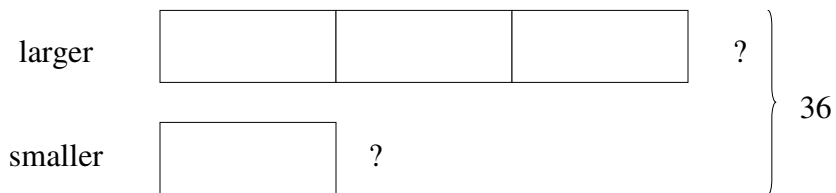
Example: I earned three times as much money this week as I did last week. Last week I earned \$150. How much more did I earn this week than last week?



3 units = 150
 1 unit = 50
 150 - 50 = 100

I earned \$100 more
 this week than last
 week

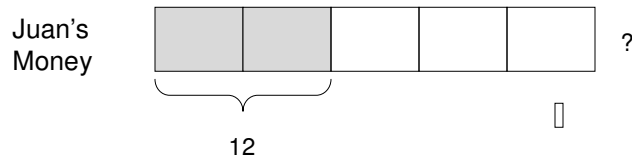
Example (grade 3): The sum of two numbers is 36. The smaller number is one-third of the larger number. Find the two numbers.



4 units = 36
 1 unit = 9
 3 units = 27

The numbers are 9 and 27.

Example (grade 4): Juan spent $\frac{2}{5}$ of his money on a CD. The CD cost \$12. How much money did he have at first?

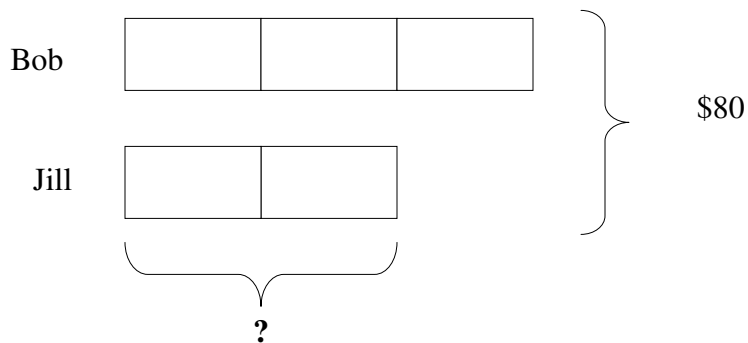


2 units = 12

1 unit = 6

5 units = 30 Juan started with \$30.

Example (grade 5): Bob and Jill share \$80 in the ratio 3:2. How much money did Bob get?



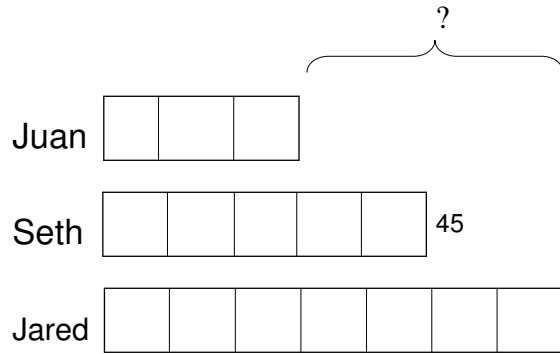
5 units = 80

3 units = 48

1 unit = 16

Bob gets \$48.

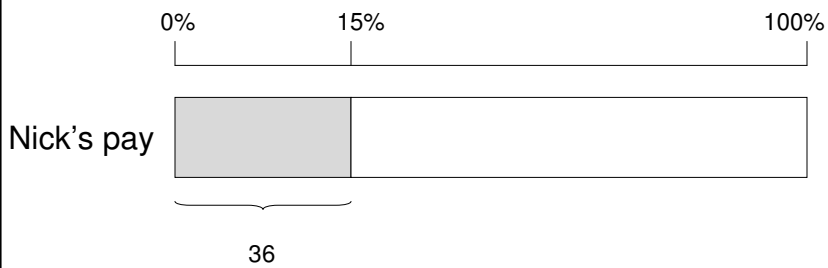
Three boys, Juan, Seth and Jared shared a number of stamps in the ratio of 3:5:7. If Seth received 45 stamps, how many more stamps did Jared receive than Juan?



Can you finish this?

2.83
0.59

Nick spent \$36 on dinner. This was 15% of his paycheck. How much was his paycheck?



$$15\% \rightarrow 36$$

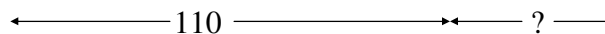
$$5\% \rightarrow 12$$

$$100\% \rightarrow 12 \times 20 = 240$$

Nick's paycheck was
\$240

Juan spent $\frac{3}{5}$ of his money in the first week and $\frac{1}{3}$ of the remainder in the second week. He spent \$110 altogether. How much did he have left?

Juan spent $\frac{3}{5}$ of his money in the first week and $\frac{1}{3}$ of the remainder in the second week. He spent \$110 altogether. How much did he have left?



11 units = \$110

4 units = \$40

1 unit = \$10

He has \$40 left.

