

# ***Problem Solving With Model Drawing***

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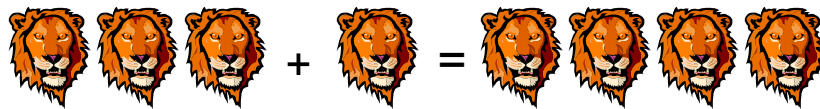
## **PROBLEM SOLVING WITH MODEL DRAWING**

The model drawing approach takes students from the concrete to the abstract stage via an intermediary pictorial stage.

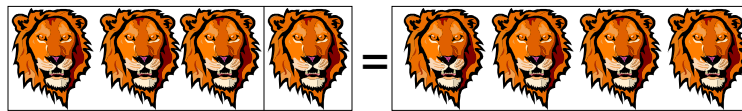
Students create bars and break them down into “units.” The units create a bridge to the concept of an “unknown” quantity that must be found.

Students can learn to use this strategy in the primary grades and continue with it through the middle grades.

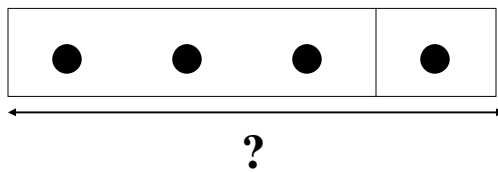
# Model Drawing Stages



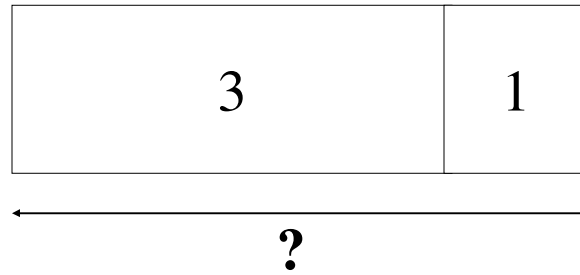
2. Draw pictures inside a bar



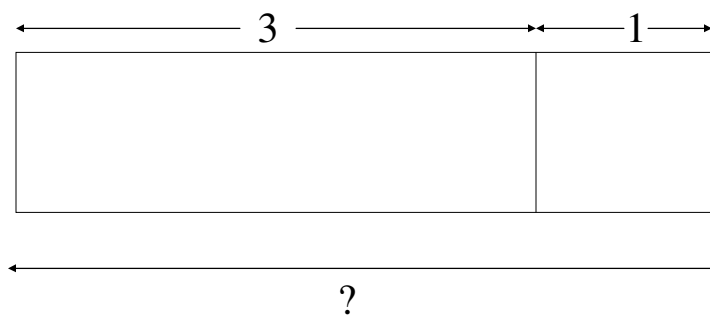
3. Replace pictures with dots and draw arrows outside bars



4. Use numbers inside the bars

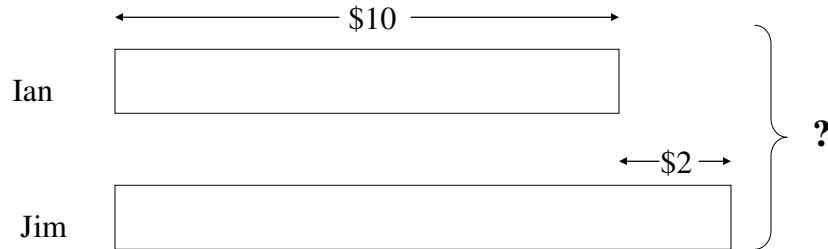


5. Use numbers outside the bars

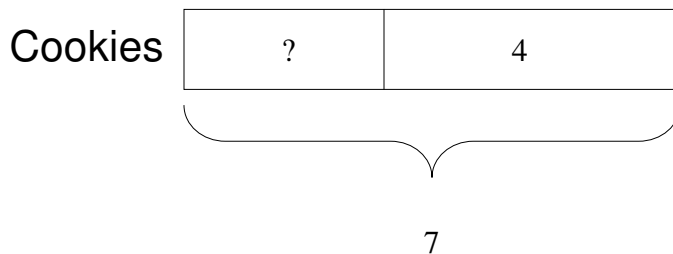


6. Label the bars.

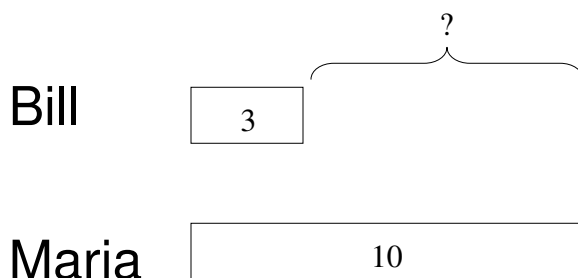
**Example:** Ian has \$10. Jim has \$2 more than Ian. How much do they have altogether.



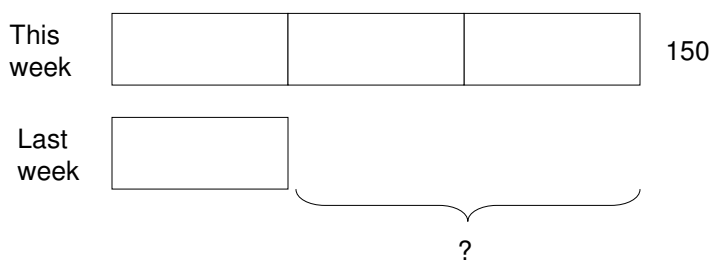
Bill has 7 cookies. He eats 4 cookies.  
How many cookies remain?



Maria has 10 cookies. Bill has 3 cookies. How many more cookies does Maria have?



**Example:** I earned three times as much money this week as I did last week. This week I earned \$150. How much more did I earn this week than last week?



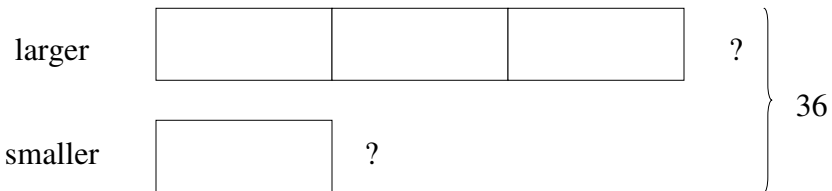
$$3 \text{ units} = 150$$

$$1 \text{ unit} = 50$$

$$150 - 50 = 100$$

I earned \$100 more  
this week than last  
week

**Example** (grade 3): The sum of two numbers is 36. The smaller number is one-third of the larger number. Find the two numbers.



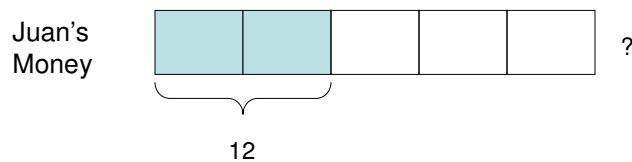
4 units = 36

1 unit = 9

3 units = 27

The numbers are 9 and 27.

**Example** (grade 4): Josh spent  $\frac{2}{5}$  of his money on a CD. The CD cost \$12. How much money did he have at first?

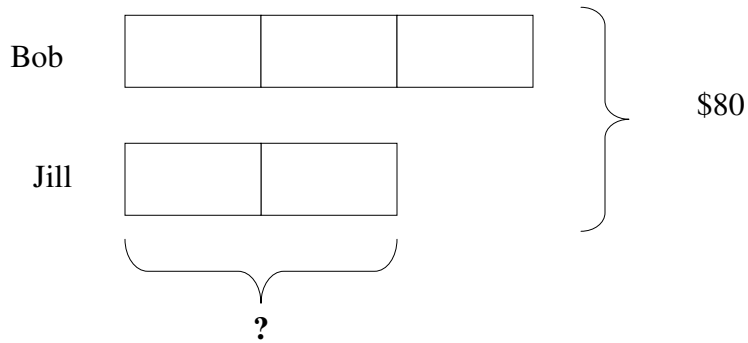


2 units = 12

1 unit = 6

5 units = 30      Juan started with \$30.

**Example** (grade 5): Bob and Jill share \$80 in the ratio 3:2. How much money did Bob get?



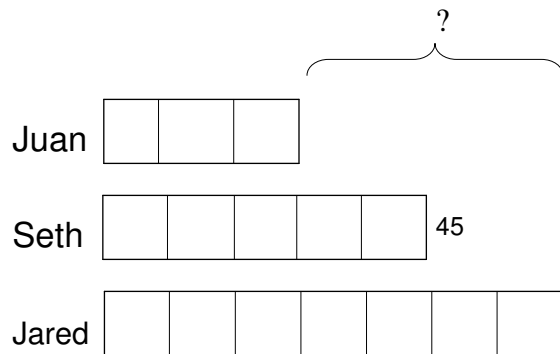
5 units = 80

3 units = 48

1 unit = 16

Bob gets \$48.

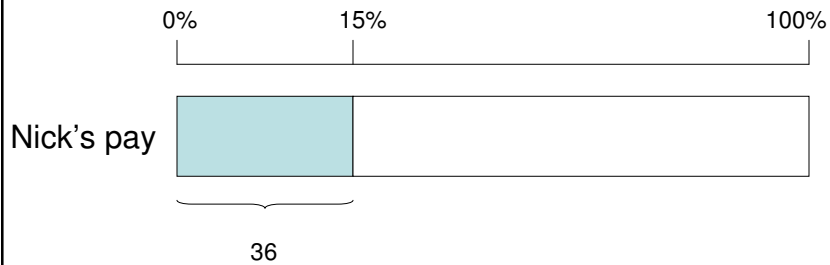
Three boys, Juan, Seth and Jared shared a number of stamps in the ratio of 3:5:7. If Seth received 45 stamps, how many more stamps did Jared receive than Juan?



Can you finish this?

2.83
0.59

Nick spent \$36 on dinner. This was 15% of his paycheck. How much was his paycheck?



$$15\% \rightarrow 36$$

$$5\% \rightarrow 12$$

$$100\% \rightarrow 12 \times 20 = 240$$

Nick's paycheck was \$240

Than spent  $\frac{3}{5}$  of his money in the first week and  $\frac{1}{3}$  of the remainder in the second week. He spent \$110 altogether. How much did he have left?



← 110 →      ← ? →



$$11 \text{ units} = \$110$$

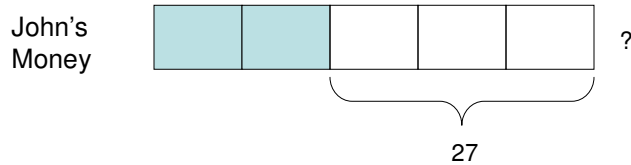
$$4 \text{ units} = \$40$$

$$1 \text{ unit} = \$10$$

**He has \$40 left.**



After spending  $\frac{2}{5}$  of his allowance on a shirt, John still has \$27 left. How much was his allowance?

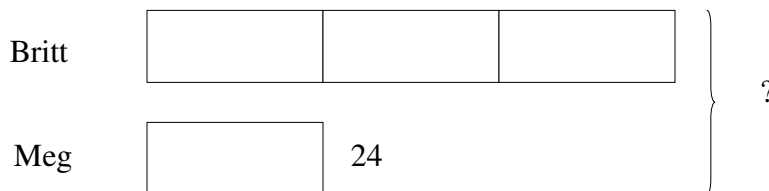


$$3 \text{ units} = 27$$

$$1 \text{ unit} = 9$$

$$5 \text{ units} = 45 \quad \text{John started with \$45.}$$

Meg has 24 stamps. Britt has 3 times as many stamps as Meg. How many do they have altogether?



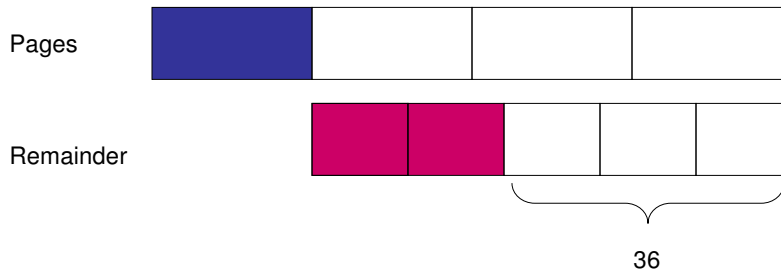
$$1 \text{ unit} = 24$$

$$4 \text{ units} = 96 \quad \text{Altogether they have 96 stamps.}$$

## More to Try - 3

- I read  $\frac{1}{4}$  of a book on Saturday and  $\frac{2}{5}$  of the remainder on Sunday. After Sunday, 36 pages remained to be read. How many pages are in the book altogether?
  
- The ratio of Ian's money to Juan's money is 2:3. After spending half of his money, Juan had \$60 less than Ian. How much money does Ian have?

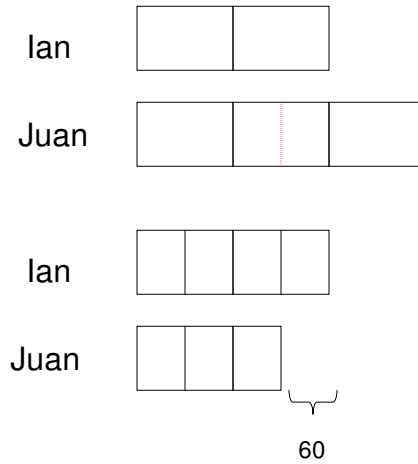
I read  $\frac{1}{4}$  of a book on Saturday and  $\frac{2}{5}$  of the remainder on Sunday. After Sunday, 36 pages remained to be read. How many pages are in the book altogether?



<b>3 units = 36</b>	<b>3 units = 60</b>
<b>1 unit = 12</b>	<b>1 unit = 20</b>
<b>5 units = 60</b>	<b>4 units = 80</b>

My book has 80 pages.

The ratio of Ian's money to Juan's money is 2:3. After spending half of his money, Juan had \$60 less than Ian. How much money does Ian have?



**References:**

*Primary Mathematics Textbooks:* Curriculum Planning and Development Division, Ministry of Education, Federal Publications, Singapore.

*A Handbook for Mathematics Teachers in Primary Schools:* Huat and Huat, Federal-Marshall Cavendish Publications, Singapore.

*Elementary Mathematics for Teachers:* Parker and Balderidge, Sefton-Ash Publishing, Okemos, Michigan.

*2005 Grade 10 Mathematics Exam:* Massachusetts Comprehensive Assessment System